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A Model with Exact Inflationary Solution in Finsler Universe

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Abstract In this paper, using the Gravity's Rainbow theory, we introduce rainbow metric into rainbow Robertson-Walker metric, and obtain a model which depends on the energy of probe particles. Furthermore, we research on an exact inflationary solution of the model, and it can be consistent with the conclusions of observation. The results of our research show that some details in inflation depend on the energy of particles which are observed by observers.

Keywords Exact inflationary solution · Rainbow-Friedmann-Robertson-Walker metric · Finsler geometry · WMAPs · Hubble factor

1 Introduction

The general relativity was put forward by Einstein in 1915, and the time and space aren't regarded as respective concepts in the theory. From then on, by several observation and experiments people have proved the results of Einstein's theory. In 1917, Einstein introduced the general relativity into the research of cosmology, and then researchers brought forward many of cosmology models to research and describe our cosmos [1–6]. In the models, the most famous and successful model is the Big Bang theory which was proposed by Gamow. However, several difficulties, such as the monopole problem and the horizon problem, haunted the research of the theory when the theory was proposed. Since the 1980s, the inflationary theory has been introduced into the Big Bang model by Guth et al., and the difficulties were resolved finally. In inflationary theory, early cosmos had an inflationary, and the inflationary was so sharp that the bulk of cosmos increases by e^{60} times or more instanter. Therefore, the difficulties of Gamow's theory could be resolved successfully. Nevertheless, there are still a large number of details needing to study.

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Recently, in the research about modern high energy physics theory and gravitational theory, researchers have put forward the gravity's rainbow theory [7–9]. In the theory, the spacetime which is observed by us depends on the energy of observed particles, so the correction Lorentz invariant of energy and momentum in flat spacetime is

$$f^2(E,\lambda)E^2 - h^2(E,\lambda)\vec{p}\bullet\vec{p} = m^2, \tag{1}$$

where, f and h are tow correction terms that correlate with energy of probe particles, and the λ and E are Planck scale and energy of probe particles. In fact, the effect from the correction terms could be found in the high energy experiments, while the effect could be ignored as $E \ll E_p$, ($E_p = 1/\sqrt{8\pi G}$ is Planck's energy), and we have used the unit $c = 1$ in the formula. According to the theory, the flat spacetime could be written as

$$ds^2 = -f^{-2}dt^2 + h^{-2}dr^2 + h^{-2}r^2 d\Omega^2. \tag{2}$$

As we all see, the flat spacetime in gravity's rainbow theory isn't Euclid spacetime, because the metric depends on the energy of probe particles, and the nature of rainbow flat spacetime is still complicated. The flat spacetime is the simplest spacetime and people are familiar with the spacetime very much. The predictability of flat gravity's rainbow theory could be found firstly, and it could strongly influence modern physics theory, if the effect is proved as reality. In this paper, we research on the rainbow Universe, and our results show the detail of inflationary could correlate with tangent vector for the energy of probe particles that can be regarded as the tangent vector in Finsler geometry.

2 Universe Metric and a Model Inflationary in Gravity's Rainbow

Generalizing the gravity's rainbow theory in the general relativity, we can get the Rainbow-Friedmann-Robertson-Walker cosmos metric [7]

$$ds^2 = -f^{-2}(E,\lambda)dt^2 + h^{-2}(E,\lambda)a^2(t)\gamma_{ij}dx^i dx^j, \tag{3}$$

where, the effect from the correction terms could be ignored, if $E \ll E_p$, and the metric could become the FRW metric, In (3), $a(t)$ is the time-dependent expansion factor, and γ_{ij} represents the spatially homogeneous (positive curvature $K = 1$, and negative curvature $K = -1$, Euclidean space $K = 0$, $\gamma_{ij} = \delta_{ij}$). According (3) and Einstein's field equations. We can obtain the formula in rainbow cosmos.

$$(\dot{a}/a)^2 = 8\pi G(E,\lambda)f^{-2}\rho/3 - Ka^{-2}h^2 f^{-2} + \Lambda(E,\lambda)/3, \tag{4}$$

$$\ddot{a}/a = -4\pi G(E,\lambda)(\rho + 3p)f^{-2}/3 + \Lambda(E,\lambda)/3. \tag{5}$$

Due to the observation, we think the space of universe is flat, so, in our model, we choose

$$K = 0, \qquad \Lambda = 0. \tag{6}$$

In the exact inflationary model in rainbow Universe, generalizing the method of Refs. [10–12], we set the scalar field and potential are $\varphi(t)$ and $V(\varphi)$, while the energy density $\rho(t)$ and pressure and $p(t)$ are

$$\rho = \frac{1}{2}\dot{\varphi}^2 + Vf^{-2}, \tag{7}$$

$$p = \frac{1}{2}\dot{\varphi}^2 - Vf^{-2}. \tag{8}$$

So the cosmos equation could be rewritten as

$$H^2 = \kappa\rho/3f^2, \tag{9}$$

$$2\dot{H} + 3H^2 = -\kappa p/f^2, \tag{10}$$

where $\kappa \equiv 8\pi G(E)$, and the Hubble factor is $H \equiv \dot{a}/a$. From the formula, we have

$$\dot{\varphi} = -\frac{2}{\kappa} f^2 H', \tag{11}$$

$$V = \frac{3}{\kappa} f^4 H^2 - \frac{2f^6}{\kappa^2}(H')^2. \tag{12}$$

We have obtained some equations, and we could get the forms of the scalar field and potential exact solution form (11) and (12), if we know the detail of $H = H(\varphi)$. Now, let's research on the inflationary model from Hubble factor's form

$$H = b\varphi^n/f^2, \tag{13}$$

where the $n > 1$. From the (11) and (13), we can get

$$\varphi = \begin{cases} (\varphi_0^{2-n} + 2bn(n-2)t/\kappa)^{1/(2-n)}, & n \neq 2, \\ \varphi_0 \exp(-4bt/k), & n = 2 \end{cases} \tag{14}$$

and from the definition of Hubble factor,

$$a = \begin{cases} a_0 \exp(\frac{\kappa}{4f^2 n}(\varphi_0^2 - (\varphi_0^{2-n} + 2bn(n-2)t/\kappa)^{\frac{2}{2-n}})), & n \neq 2, \\ a_0 \exp(\kappa\varphi_0^2(1 - e^{-8bt/\kappa})/8f^2), & n = 2 \end{cases} \tag{15}$$

where $a_0 = a(t)|_{t=0}$ is the initial expansion factor. According to (12) and (13), we have

$$V = 3b^2\varphi^{2n}/\kappa - 2b^2n^2 f^2\varphi^{2n-2}/\kappa^2. \tag{16}$$

Now, we have obtained a model with exact inflationary solution with $H = b\varphi^n/f^2$ in rainbow universe, and we find expansion factor and potential are probing particles' energy dependent, and the effect could be proved in future astronomic observation. In next section, we will restrict the model with the data of WMAP5.

3 Discussion

In the research of inflation theory, several parameters are very crucial,

$$\varepsilon(\varphi) \equiv \frac{2}{\kappa}\left(\frac{H'(\varphi)}{H(\varphi)}\right)^2, \tag{17}$$

$$\eta(\varphi) \equiv \frac{2}{\kappa}\left(\frac{H''(\varphi)}{H(\varphi)}\right), \tag{18}$$

$$\xi(\varphi) \equiv \frac{2}{\kappa}\left[\frac{H'''(\varphi)H'(\varphi)}{H^2(\varphi)}\right]^{1/2} \tag{19}$$

we set the scalar field φ_f is the value when the inflationary finishes, and from the inflationary theory, the formula should be satisfied

$$\varepsilon(\varphi_f) = 1. \tag{20}$$

Therefore, from (20) and (17), we get the scalar field φ_f

$$\varphi_f = \sqrt{2n^2/\kappa}. \tag{21}$$

Considering the fact that the effect from the correction terms is a little correction and could be ignored, if $E \ll E_p$, let's study the case of $f = h = 1$firstly. The number of e-folds of inflation is

$$N \equiv \ln \frac{a_e}{a_f} = \int_{\varphi}^{\varphi_f} \frac{H}{\dot{\varphi}} d\varphi = \frac{\kappa}{4n}(\varphi^2 - \varphi_f^2). \tag{22}$$

We must have $N > 60$, if our model could inflate successfully, and tally with the observation today, so we require at least

$$N = \frac{\kappa}{4n}(\varphi_{60}^2 - \varphi_f^2) = 60. \tag{23}$$

From the data of WMAP5 [13] $n_s = 0.960 \pm 0.013$ and the formula

$$1 - n_s = 4\varepsilon - 2\eta + 8(1+c)\varepsilon^2 - 2(3+5c)\varepsilon\eta + 2c\varepsilon\xi, \tag{24}$$

where, $c = -2 + \ln 2 + \gamma(\gamma \approx 0.557$ is Euler constant), the range of n is

$$n \in [0.629, 2.188] \tag{25}$$

in the range, our model is reasonable. However, we can still widen the range due to the effect of rainbow correction terms.

In this paper, we have researched on an exact rainbow inflationary model, and the conclusion shows that the detail of inflation could depend on the probe particles' energy [14, 15]. Considering the metric of rainbow-FRW is energy dependent, the spacetime is a Finsler spacetime. In geometry theory, Finsler metric is tangent vector dependent, and the Finsler geometry is general Riemannian geometry. Therefore, from the viewpoint of differential geometry, the Riemannian geometry couldn't depict all the nature of Finsler geometry, and the Einstein's theory, which builds on the Riemannian geometry, couldn't get the detail of correction terms. What we need is a new gravity theory which builds on the Finsler geometry, and the research about Finsler gravity theory could make physics into a new era.

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